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Research Methodology

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Testing of hypothesis.

The steps in testing of hypothesis are —

1) State the null hypothesis (H_0) and alternative hypothesis (H_1)

2) State the level of significance, a (alpha) for the test

3) Establish critical or rejection region

4) Calculate the suitable test statistic.

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HYPOTHESIS TESTING

A statistical hypothesis is a claim (belief or assumption) about an unknown population parameter value. The methodology that enables a decision-maker to draw inferences about population characteristics by analysing the difference between the value of sample statistic and the corresponding hypothesized parameter value, is called *hypothesis testing*.

A General Procedure for Hypothesis Testing

To test the validity of the claim or assumption about the population parameter,a sample is drawn from the population and analysed. The results of the analysis are used to decide whether the claim is true or not. The steps of general procedure for any hypothesis testing are summarized below.

Step 1: State the null hypothesis (H_0) and alternative hypothesis (H_1)

The null hypothesis H_0 refers to a hypothesized numerical value or range of values of the population parameter. Theoretically hypothesis testing requires that the null hypothesis be considered true until it is proved false on the basis of results observed from the sample data. The null hypothesis is always expressed in the form of an equation making a claim regarding the specific value of the population parameter. This is :

$$H_0 : \mu = \mu_0$$

where μ is population mean and μ_0 represents hypothesized parameter value.

An alternative hypothesis, H_1, is the logical opposite of the null hypothesis, that is, an alternative hypothesis must be true when the null hypothesis is found to be false. In other words, the alternative hypothesis states that specific population parameter value is not equal to the value stated in the null hypothesis and is written as :

$$H_1 : \mu \neq \mu_0$$

Consequently, $\quad H_1 : \mu < \mu_0$ or $H_1 : \mu > \mu_0$

Step 2: State the level of significance, α (alpha) for the test

The level of significance, usually denoted by α (alpha), is specified before the samples are drawn, so that the results obtained should not influence the choice of the decision-maker. It is specified in terms of the level of probability of null hypothesis H_0 being wrong. In other words, a probability which has a null hypothesis may be rejected when it is true.

Step 3: Establish critical or rejection region

Sample space of the experiment which corresponds to the area under the sampling distribution curve of the test statistic is divided into two mutually exclusive regions as shown in Fig. 4.9.These regions are called the acceptance region and the rejection or critical region.

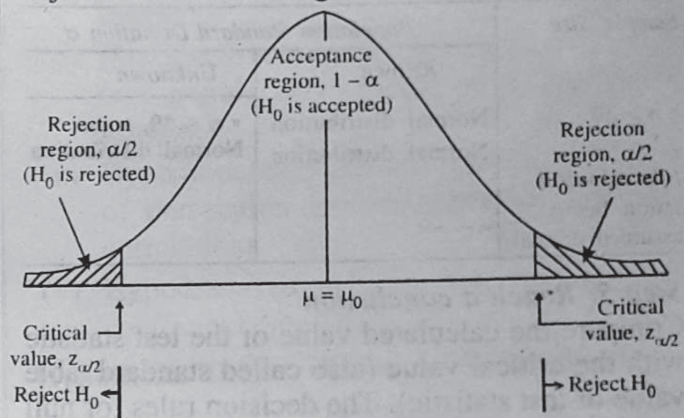

Fig. 4.9 : Areas of Acceptance and Rejection of H_0 (Two-Tailed Test)

If the value of the test statistic falls into the acceptance region, the null hypothesis is accepted, otherwise it is rejected. The rejection region consists of all values of the test satistic that are likely to occur if null hypothesis is true. On the other hand, these values are not likely to occur if null hypothesis is false. The value of the sample statistic that separates the regions of acceptance and rejection is called critical value.

The size of the rejection region is directly related to the level of precision to make decision about a population parameter. Decision rules concerning null hypothesis are as follows :

- Prob (H_0 is true) $\leq \alpha$, reject H_0
- Prob (H_0 is true) $> \alpha$, accept H_0

In other words, if probability of H_0 being true is less than α, reject H_0, otherwise retain H_0.

Step 4: Calculate the suitable test statistic

The value of test statistic is calculated from the distribution of sample statistic by using the following formula

$$\text{Test statistic} = \frac{\text{Value of sample statistic} - \text{Value of hypothesized population parameter}}{\text{Standard error of the sample statistic}}$$

For example, $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$; σ is known, $n > 30$

The choice of a probability distribution of a sample statistic is guided by the sample size n and the value of population standard deviation σ as shown in Table 4.4.

Table 4.4 : Choice of Probability Distribution

Sample Size	*Population Standard Deviation σ*	
	Known	*Unknown*
• $n > 30$	Normal distribution Normal distribution	• $n \leq 30$, popu- Normal distribution
t-distribution lation being assumed normal		

Step 5: Reach a conclusion

Compare the calculated value of the test statistic with the critical value (also called standard table value of test statistic). The decision rules for null hypothesis are as follows :

- $|z|_{cal} \geq |z|_{Table}$; reject H_0
- $|z|_{cal} < |z|_{Table}$; accept H_0

In other words, if the calculated absolute value of a test statistic is more than or equal to its critical (or table) value, then reject the null hypothesis, otherwise accept it.

One-tailed and two-tailed tests

There are two types of tests of hypotheses, referred to as the one-tailed and two-tailed tests. The type of tests used depends upon the way the hypotheses are formulated.

(i) Null hypothesis and alternative hypothesis stated as
$H_0 : \mu = \mu_0$ and $H_1 : \mu \neq \mu_0$
imply that any deviation (either on the lower side or higher side) of the calculated value of test statistic from a hypothesized value μ_0 leads to rejection of the null hypothesis. Hence, it is necessary to keep the rejection region on 'both tails' of the sampling distribution of the test statistic. This type of test is called two-tailed test as shown in Fig. 4.9. If the significance level for the test is α per cent, then rejection region equal to α 2 per cent is kept in each tail of the sampling distribution.

(ii) Null hypothesis and alternative hypothesis stated as
$H_0 : \mu \leq \mu_0$ and $H_1 : \mu > \mu_0$ (Right-tailed test)
or
$H_0 : \mu \geq \mu_0$ and $H_1 : \mu < \mu_0$ (Left-tailed test)
imply that the value of sample statistic is either 'higher' or 'lower' than the hypothesized parameter value. This leads to the rejection of null hypothesis for significant deviation from the specified value μ_0 in one direction (or tail) of the curve of sampling distribution. Thus, the entire rejection region corresponding to the level of significance, α per cent, is in only one tail of the sampling distribution of the statistic, as shown in Fig. 4.10(a) and (b).

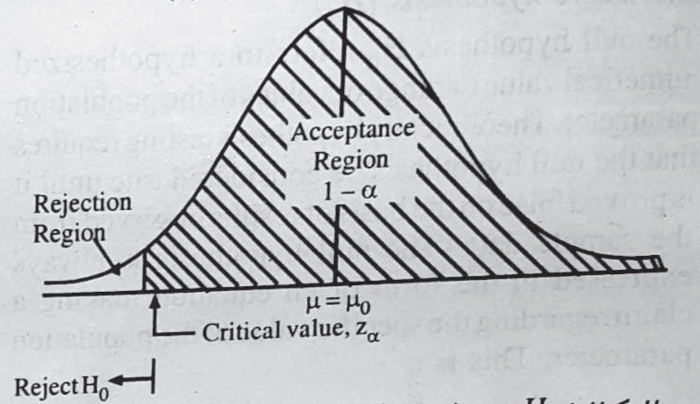

Fig. 4.10(a) : Left-tailed, $H_0 : \mu \geq \mu_0$; $H_1 : \mu < \mu_0$

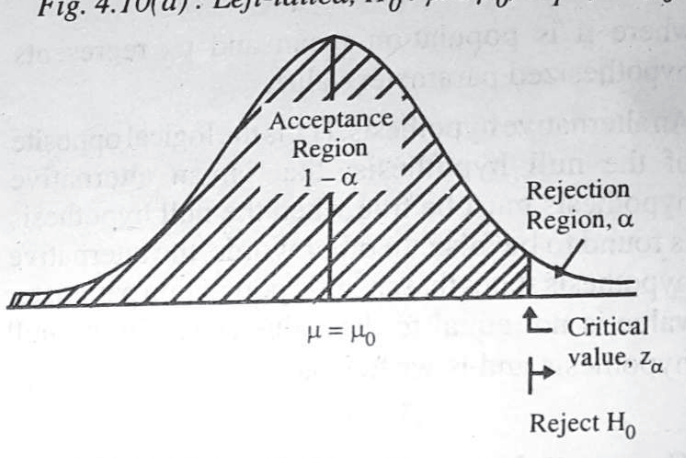

Fig. 4.10(b): Right-tailed Test, $H_0 : \mu \leq \mu_0$; $H_1 : \mu > \mu_0$

A summary of certain critical values at various significance levels for test statistic z is given in Table 4.5.

Table 4.5 : Summary of Certain Critical Values for Sample Statistic z

Rejection Region	*Level of Significance, α per cent*				
	10%	*5%*	*1%*	*0.5%*	*0.02%*
One-tailed region	±1.28	±1.64	±2.33	±2.58	±2.88
Two-tailed region	±1.645	±1.96	±2.58	±2.81	±3.08